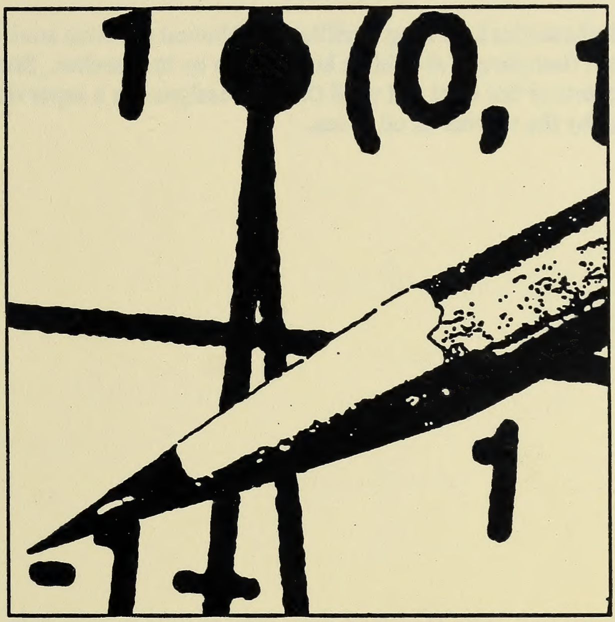
MATHEMATICS 3

LEARNING FACILITATOR'S MANUAL



UNIT 8: SYSTEMS OF LINEAR EQUATIONS

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final test until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 31
Learning Facilitator's Manual
Unit 8
Systems of Linear Equations
Alberta Distance Learning Centre
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Topic 1: Methods of Solving Systems of Linear Equations with Two Variables

5 1. Solve the following system of equations.

$$\frac{x}{2}-\frac{y}{5}=\frac{1}{10} \quad (1)$$
$$5x-y=10 \quad (2)$$

Write (2) as follows:

$$y=5x-10 \quad (3)$$

Substitute (3) in (1).

$$\frac{x}{2}-\frac{(5x-10)}{5}=\frac{1}{10}$$
$$\frac{x}{2}-(x-2)=\frac{1}{10}$$
$$\frac{x-2(x-2)}{2}=\frac{1}{10}$$
$$10(x-2x+4)=2$$
$$-10x+40=2$$
$$-10x=-38$$
$$x=\frac{38}{10}$$
$$=3.8$$

Substitute $x=3.8$ in (3).

$$y=5(3.8)-10$$
$$=19-10$$
$$=9$$

The solution is $x=3.8$ and $y=9$.

5 2. Use the elimination method to solve the following system of equations.

$$5x-3y=8 \quad (1)$$
$$3x+5y=12 \quad (2)$$

$$\begin{aligned} 5\times(1)&: & 25x-15y&=40 & (3)\\ 3\times(2)&: & 9x+15y&=36 & (4)\\ (3)+(4)&: & 34x&=76 &\\ & & x&=\frac{76}{34} &\\ & & &=\frac{38}{17} & \end{aligned}$$

Substitute $x=\frac{38}{17}$ in (2).

$$3\left(\frac{38}{17}\right)+5y=12$$
$$5y=12-\frac{114}{17}$$
$$5y=\frac{204}{17}-\frac{114}{17}$$
$$5y=\frac{90}{17}$$
$$y=\frac{90}{85}$$
$$y=\frac{18}{17}$$

The solution is $x=\frac{38}{17}$ and $y=\frac{18}{17}$.

10

3. Solve the following systems of equations, and identify each system of equations as dependent, independent, or inconsistent.

a. $3x+8y=12$ (1)

$2x+\frac{16}{3}y=8$ (2)

$$\begin{array}{lrl} 2\times(1): & 6x+16y=24 & (3) \\ 3\times(2): & 6x+16y=24 & (4) \\ \hline (3)-(4): & 0=0 & \end{array}$$

Let $y=k$.

$3x+8k=12$

$3x=12-8k$

$x=\frac{12-8k}{3}$

Therefore, the solution set is $\left\{\left(\frac{12-8k}{3}, k\right) \middle| k \in R\right\}$.

The system is a dependent system.

b. $5x-3y=6$ (1)

$10x-6y=10$ (2)

$$\begin{array}{lrl} & 10x-6y=10 & (2) \\ 2\times(1): & 10x-6y=12 & (3) \\ \hline (2)-(3): & 0=-2 & \end{array}$$

Therefore, the solution set is ϕ.
The system of equations is an inconsistent system.

5

4. Solve the following system of equations. State the solution set, and sketch the graph.

$3x - y = 4$ ①
$x + 8y = 43$ ②

① $- 3\times$②: $-25y = -125$
$y = 5$

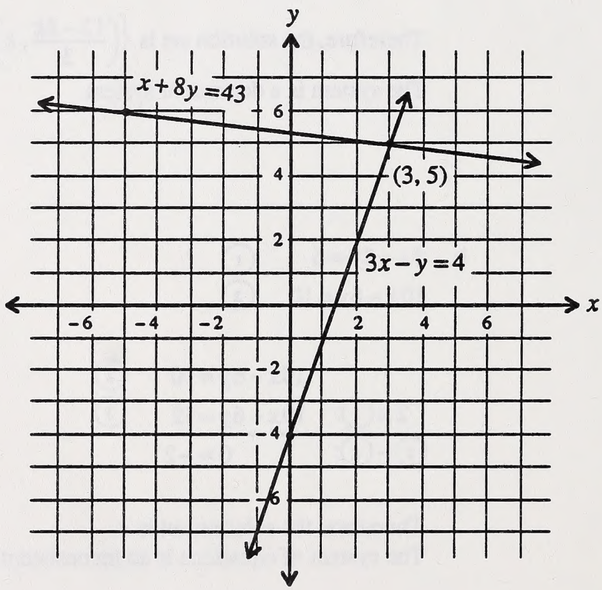
Substitute $y = 5$ in ①.

$x = 43 - 8y$
$= 43 - 40$
$= 3$

Therefore, the solution set is $\{(3, 5)\}$.

For $3x - y = 4$:

x	3	0
y	5	-4



For $x + 8y = 43$:

x	3	-5
y	5	6

5

5\. a. Identify the following systems of equations as dependent, independent, or inconsistent.

i. $\left.\begin{array}{l} x-3y=7 \\ x-3y=2 \end{array}\right\}$ **Inconsistent**

ii. $\left.\begin{array}{l} 2x+y=4 \\ 4x+2y=8 \end{array}\right\}$ **Dependent**

iii. $\left.\begin{array}{l} 2x-3y=5 \\ 5x+2y=-3 \end{array}\right\}$ **Independent**

b. Solve iii.

$$2x-3y=5 \quad ①$$
$$5x+2y=-3 \quad ②$$

$$\begin{array}{rl} 2\times①: & 4x-6y=10 \\ 3\times②: & 15x+6y=-9 \\ \hline ③+④: & 19x=1 \\ & x=\frac{1}{19} \end{array}$$

Substitute $x=\frac{1}{19}$ in ①.

$$2\left(\frac{1}{19}\right)-3y=5$$
$$-3y=5-\frac{2}{19}=4\frac{17}{19}$$
$$y=-\frac{93}{3(19)}=\frac{-31}{19}$$

Therefore, $x=\frac{1}{19}$ and $y=-\frac{31}{19}$.

Topic 1

_______ marks

Topic 2: Systems of Linear Equations with Three Variables

6

1. Solve the following system of equations.

$$3x-8y+z=-10 \quad (1)$$
$$-5x+y-6z=-13 \quad (2)$$
$$x-y+2z=5 \quad (3)$$

$$(1)-3\times(3): \quad -5y-5z=-25 \quad (4)$$
$$(2)+5\times(3): \quad -4y+4z=12 \quad (5)$$

$$4\times(4)+5\times(5): \quad -40y=-40$$
$$y=1$$

Substitute $y=1$ into (4).

$$-5-5z=-25$$
$$-5z=-20$$
$$z=4$$

Substitute $y=1$ and $x=4$ into (3).

$$x-1+2(4)=5$$
$$x=-2$$

The solution set is $\{(-2,1,4)\}$.

8

2. Solve the following system of equations, and classify the system as dependent, independent, or inconsistent.

$$2x-y+4z=15 \quad (1)$$
$$x-2y+3z=5 \quad (2)$$
$$3x-9y+11z=10 \quad (3)$$

$$\begin{aligned} (1)-2\times(2)&: & 3y-2z&=5 & (4)\\ (3)-3\times(2)&: & -3y+2z&=-5 & (5)\\ \hline (4)+(5)&: & 0y+0z&=0 \end{aligned}$$

Let $z=k$.

The following can be stated from (4):

$$\begin{aligned} 3y-2k&=5\\ 3y&=2k+5\\ y&=\frac{2k+5}{3} \end{aligned}$$

The following can be stated from (2):

$$\begin{aligned} x-2\left(\frac{2k+5}{3}\right)+3k&=5\\ x-\frac{4k+10}{3}+3k&=5\\ x-\frac{4k+10}{3}+\frac{9k}{3}&=5\\ x&=5-\frac{5k-10}{3}\\ &=\frac{25-5k}{3} \end{aligned}$$

The solution set is $\left\{\left(\frac{25-5k}{3}, \frac{2k+5}{3}, k\right) \middle| k \in R\right\}$.

The system is a dependent system.

4

3. a. Define a linear homogeneous equation.

A linear homogeneous equation contains first-degree variables and a constant term of zero.

b. Show that the following expression is linear and homogeneous.

$$\frac{1}{3}x-\frac{2}{5}y+6-y^2=\frac{3}{5}y+6-2z-\frac{2}{3}x-y^2$$

$$\frac{x}{3}-\frac{2y}{5}+6-y^2=\frac{3y}{5}+6-2z-\frac{2x}{3}-y^2$$

$$\frac{x}{3}+\frac{2x}{3}-\frac{2y}{5}-\frac{3y}{5}+6-6-y^2+y^2+2z=0$$

$$\frac{3x}{3}-\frac{5y}{5}+0+0+2z=0$$

$$x-y+2z=0$$

This is a linear homogeneous equation.

7

4. Solve the following nonhomogeneous system of equations.

$$3x-6y+z=50 \quad (1)$$
$$x+4y-3z=8 \quad (2)$$

$(1)-3\times(2)$: $-18y+10z=26 \quad (3)$

Let $z=k$.

$$\begin{aligned}-18y+10k&=26\\ -18y&=-10k+26\\ y&=\frac{10k-26}{18}\\ &=\frac{5k-13}{9}\end{aligned}$$

Substitute for z and y in (2).

$$\begin{aligned}x+4\left(\frac{5k-13}{9}\right)-3k&=8\\ x+\frac{20k-52}{9}-\frac{27k}{9}&=8\\ x+\frac{-7k-52}{9}&=8\\ x&=\frac{7k+52}{9}+8\\ &=\frac{7k+124}{9}\end{aligned}$$

The solution set is $\left\{\left(\frac{7k+124}{9}, \frac{5k-13}{9}, k\right) \middle| k \in R\right\}$.

Topic 2

________ marks

Topic 3: Matrices – Definitions and Properties

2

1. Write the augmented matrix for the following system of equations.

$$\left.\begin{aligned} x-y-3z&=5 \\ 2x+y-7z&=8 \\ x-3y+5z&=9 \end{aligned}\right\}$$

$$\begin{bmatrix} 1 & -1 & -3 & 5 \\ 2 & 1 & -7 & 8 \\ 1 & -3 & 5 & 9 \end{bmatrix}$$

8 2. Find a row-reduced echelon form for the following matrix. (Be sure to show all steps.)

$$\begin{bmatrix} 5 & 1 & -3 \\ 4 & 0 & -1 \\ 1 & -2 & 6 \end{bmatrix}$$

$$\begin{array}{cc} \begin{bmatrix} 5 & 1 & -3 \\ 4 & 0 & -1 \\ 1 & -2 & 6 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$\begin{array}{rc} \begin{array}{r} \\ \\ 2-4\times 3: \end{array} & \begin{bmatrix} 5 & 1 & -3 \\ 4 & 0 & -1 \\ 0 & 8 & -25 \end{bmatrix} \end{array}$$

$$\begin{array}{rc} \begin{array}{r} \\ \frac{1}{4}\times 2: \\ \frac{1}{8}\times 3: \end{array} & \begin{bmatrix} 5 & 1 & -3 \\ 1 & 0 & -\frac{1}{4} \\ 0 & 1 & -\frac{25}{8} \end{bmatrix} \end{array}$$

$$\begin{array}{rc} \begin{array}{r} \\ 1-5\times 2: \\ \\ \end{array} & \begin{bmatrix} 5 & 1 & -3 \\ 0 & 1 & -1\frac{3}{4} \\ 0 & 1 & -\frac{25}{8} \end{bmatrix} \end{array}$$

$$\begin{array}{rc} \begin{array}{r} \\ \\ 2-3: \end{array} & \begin{bmatrix} 5 & 1 & -3 \\ 0 & 1 & -\frac{7}{4} \\ 0 & 0 & \frac{11}{8} \end{bmatrix} \end{array}$$

$$\begin{array}{rc} \begin{array}{r} \frac{1}{5}\times 1: \\ \\ \frac{8}{11}\times 3: \end{array} & \begin{bmatrix} 1 & \frac{1}{5} & -\frac{3}{5} \\ 0 & 1 & -\frac{7}{4} \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

4

3. Are the following two matrices row equivalent?

$$T_1 = \begin{bmatrix} 5 & 3 & 1 \\ 4 & 0 & 6 \\ 1 & 2 & 5 \end{bmatrix} \qquad T_2 = \begin{bmatrix} 5 & 3 & 1 \\ \frac{1}{2} & 0 & \frac{3}{4} \\ 0 & 8 & 14 \end{bmatrix}$$

Yes, the two matrices are row equivalent.

$$\begin{array}{r} \\ \frac{1}{8}\times (2): \\ 4\times (3) - (2): \end{array} \begin{bmatrix} 5 & 3 & 1 \\ \frac{1}{2} & 0 & \frac{3}{4} \\ 0 & 8 & 14 \end{bmatrix}$$

5

4. Successively perform the following operations to find a matrix equivalent to the given one.

$$\begin{array}{c} \begin{bmatrix} 2 & 5 & 3 \\ 1 & 4 & -2 \\ -1 & 0 & -6 \end{bmatrix} \end{array} \begin{array}{c} ① \\ ② \\ ③ \end{array}$$

a. Add row 2 to row 3.

$$\begin{array}{r} \\ \\ ②+③: \end{array} \begin{bmatrix} 2 & 5 & 3 \\ 1 & 4 & -2 \\ 0 & 4 & -8 \end{bmatrix}$$

b. Subtract row 1 from 2 times row 2.

$$\begin{array}{r} \\ 2\times②-①: \\ \\ \end{array} \begin{bmatrix} 2 & 5 & 3 \\ 0 & 3 & -7 \\ 0 & 4 & -8 \end{bmatrix}$$

c. Multiply row 1 by $\frac{1}{2}$, row 2 by $\frac{1}{3}$, and row 3 by $\frac{1}{4}$.

$$\begin{array}{r} \frac{1}{2}\times①: \\ \frac{1}{3}\times②: \\ \frac{1}{4}\times③: \end{array} \begin{bmatrix} 1 & \frac{5}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{7}{3} \\ 0 & 1 & -2 \end{bmatrix}$$

d. Subtract row 2 from row 3.

$$③-②:\begin{bmatrix} 1 & \frac{5}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{7}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

e. Multiply row 3 by 3.

$$3\times③:\begin{bmatrix} 1 & \frac{5}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{7}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

③ 5. Answer the following about the given matrix.

$$\begin{bmatrix} 3 & -2 & 1 & 5 \\ 4 & -2 & 3 & 0 \end{bmatrix}$$

a. Write the system of equations.

$$3x-2y+z=5$$
$$4x-2y+3z=0$$

b. Determine the number of rows and columns.

There are two rows and four columns.

9

6. Solve the following system of equations using **row-reduced echelon form.** (No marks will be given for any other method.)

$$\left.\begin{aligned} x-5y&=7 \\ 3x+y&=12 \end{aligned}\right\}$$

$$\begin{bmatrix} 1 & -5 & 7 \\ 3 & 1 & 12 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

$$②-3\times①: \begin{bmatrix} 1 & -5 & 7 \\ 0 & 16 & -9 \end{bmatrix}$$

$$\tfrac{1}{16}\times②: \begin{bmatrix} 1 & -5 & 7 \\ 0 & 1 & -\tfrac{9}{16} \end{bmatrix}$$

From ② you can state the following:

$$y=-\frac{9}{16}$$

From ① you can state the following:

$$\begin{aligned} x-5y&=7 \\ x&=7+5y \\ &=7+5\left(-\frac{9}{16}\right) \\ &=7-\frac{45}{16} \\ &=\frac{67}{16} \text{ or } 4\frac{3}{16} \end{aligned}$$

Therefore, the solution is $x=\frac{67}{16}$ and $y=-\frac{9}{16}$.

3

7. Identify the system of equations represented by each of the following matrices as dependent, independent, or inconsistent.

a. $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix}$

This matrix is independent.

b. $\begin{bmatrix} 1 & 4 & 9 \\ 0 & 0 & 1 \end{bmatrix}$

This matrix is inconsistent.

c. $\begin{bmatrix} 1 & -2 & 7 \\ 0 & 0 & 0 \end{bmatrix}$

This matrix is dependent.

9 8. Solve for x, y, and z in the following system of equations using the row-reduced echelon form of the augmented matrix.

$$\left.\begin{aligned} 3x-9y+z&=-43 \\ x-3y-2z&=-33 \\ 2x+y+4z&=33 \end{aligned}\right\}$$

$$\begin{bmatrix} 3 & -9 & 1 & -43 \\ 1 & -3 & -2 & -33 \\ 2 & 1 & 4 & 33 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$\begin{matrix} \\ 1-3\times 2: \\ 3-2\times 2: \end{matrix} \begin{bmatrix} 3 & -9 & 1 & -43 \\ 0 & 0 & 7 & 56 \\ 0 & 7 & 8 & 99 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{3}\times 1: \\ 2 \leftrightarrow 3: \\ \\ \end{matrix} \begin{bmatrix} 1 & -3 & \frac{1}{3} & -\frac{43}{3} \\ 0 & 7 & 8 & 99 \\ 0 & 0 & 7 & 56 \end{bmatrix}$$

$$\begin{matrix} \\ \frac{1}{7}\times 2: \\ \frac{1}{7}\times 3: \end{matrix} \begin{bmatrix} 1 & -3 & \frac{1}{3} & -\frac{43}{3} \\ 0 & 1 & \frac{8}{7} & \frac{99}{7} \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

Now from row 3 it can be stated that $z=8$.

Substitute $z=8$ in $y+\frac{8}{7}z=\frac{99}{7}$.

$$
\begin{aligned}
y+\frac{64}{7}&=\frac{99}{7}\\
y&=\frac{99}{7}-\frac{64}{7}\\
&=\frac{35}{7}\\
&=5
\end{aligned}
$$

Substitute $z=8$ and $y=5$ in $x-3y+\frac{1}{3}z=-\frac{43}{3}$.

$$
\begin{aligned}
x-3(5)+\frac{1}{3}(8)&=-\frac{43}{3}\\
x-15+\frac{8}{3}&=-\frac{43}{3}\\
x&=-\frac{43}{3}+\frac{37}{3}\\
&=-\frac{6}{3}\\
&=-2
\end{aligned}
$$

The solution is $x=-2$, $y=5$, and $z=8$.

2

9. If the row-reduced echelon form contains all zeros in the bottom row, how do you determine the solution?

   Let one variable be any arbitrary real number and calculate the other variables in terms of the arbitrary real number.

Topic 3

_______ marks

This booklet cannot be purchased separately; the Learning Facilitator's Manual is available only as a complete set.

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